Analysis of Stripline Filled with a Multilayered Dielectric Medium

K. V. SESHAGIRI RAO, MEMBER, IEEE, N. ADISESHU, MEMBER, IEEE, AND B. N. DAS

Abstract **—An analysis has been carried out using conformal transformation for the case of a symmetric stripline filled with multilayered discrete dielectric media with their interfaces parallel to the ground planes. This has been applied to a stripline structure filled with a medium having a relative dielectric constant which is continuously varying in a direction perpendicular to the ground planes. Eight different distributions, such as Gaussian, parabolic, and semicircular, are considered for this variation of relative dielectric constant. Impedance data on stripline are presented for all these cases. The results of limiting cases obtained from this general formulation are compared with those existing in the literature.**

I. INTRODUCTION

ANALYSIS OF shielded microstrip line filled with layered dielectric media has been carried out by Yamashita and Atsuki from the determination of the Green's function satisfying the boundary condition [1] and by Mittra using the function theoretic technique [2]. The number of layers of dielectrics for which the analysis was carried out was restricted to three, presumably in view of the complication involved in the derivation of analytical expressions. Joshi *et al.* used conformal transformation to find the characteristic impedance of a strip on a dielectric slab arbitrarily located parallel to and between two ground planes filled with only two dielectric layers [3]. Callarotti and Gallo presented the calculation for the capacitance and effective dielectric constant for a microstrip line with two different dielectrics using conformal transformation and the finite-difference method [4]. Cao Wei *et al.* formulated the characteristics of multiconductor dielectric media where the number of conductors and the number of dielectric layers are arbitrary. Their formulation is based on the method of moments using pulses for expansion and point matching for testing [5]. Later, they calculated the losses in the above transmission structure [6]. Subsequently, Koul reported an analytical method for the capacitance of a rectangular inhomogeneous coaxial line with offset inner conductor with anisotropic dielectric. This method involved the spectral-domain technique in discrete Fourier variable under quasi-static approximation [7]. An iterative moment method was suggested by Sultan and Mittra for calculating the electromagnetic field distribution inside inhomogeneous lossy dielectric objects [8]. Marques *et al.* developed a recurrence relation for finding out the Green's function for an arbitrary anisotropic N-layered dielectric structure [9]. However, studies on stripline characteristics with different layered dielectrics whose relative dielectric constants vary in the form of such distributions as Gaussian and inverted Gaussian have, to the best of the authors' knowledge, not been carried out, even for the quasi-static approximated case, i.e., for lower frequency ranges. It will, therefore, be of interest to study the effect of a large number of dielectric layers between the ground planes, from which the limiting case of continuous variation of dielectric constant from one ground plane to the other can be estimated.

In the present work, a method of conformal transformation is used for the analysis of a symmetric stripline filled with a large number of layers having different dielectric constants. The analysis is based on quasi-TEM approach and is strictly valid at lower frequency ranges only. The formulation is carried out from the transformation of one half of the structure, symmetric with respect to the line perpendicular to the center of the strip conductor to a parallel plate configuration [10]. In this case, a closed-form analytical expression which gives the transformations of the dielectric interfaces in the transformed parallel-plate configuration can be found. These dielectric interfaces appear in the form of curved contours. An infinitesimally small elementary column perpendicular to the parallel plate can be regarded as a series combination of a number of elementary capacitances. The capacitance of the structure can, therefore, be obtained from the parallel combination of such elementary capacitances. In the limit of vanishingly small width of the elementary column, the capacitance can be obtained in the form of an integral whose integrand contains a series summation of a function representing the contours of the transformed interfaces.

The general expression obtained here for the capacitance for the case of a number of discrete dielectric layers is further extended to the case of a stripline filled with a medium in which the variation of the relative dielectric constant is represented by a continuous function. In this case, the minimum number of dielectric layers to be considered to realize the continuous variation is obtained such that the characteristic impedance of the stripline remains constant for further increase of the dielectric layers. This minimum number of discrete layers which realizes the

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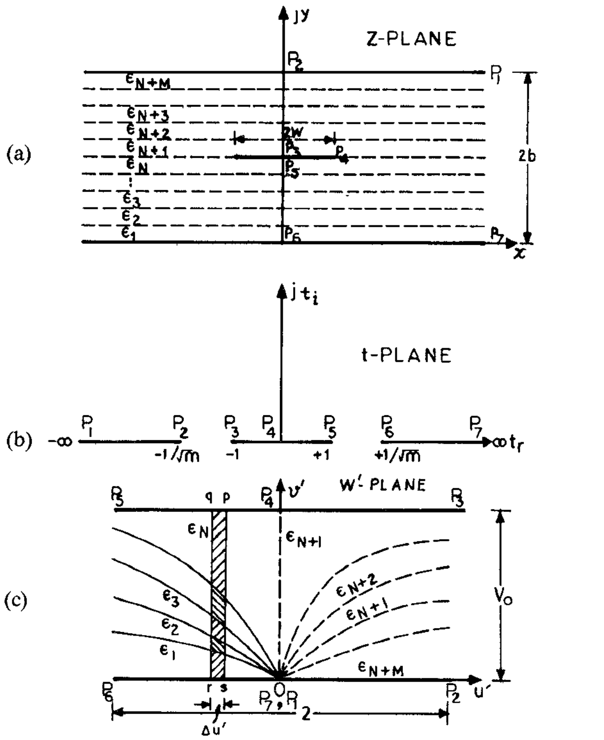

Fig. 1. Stripline filled with multilayered dielectric medium ($N+M$ layers) and its conformal representations.

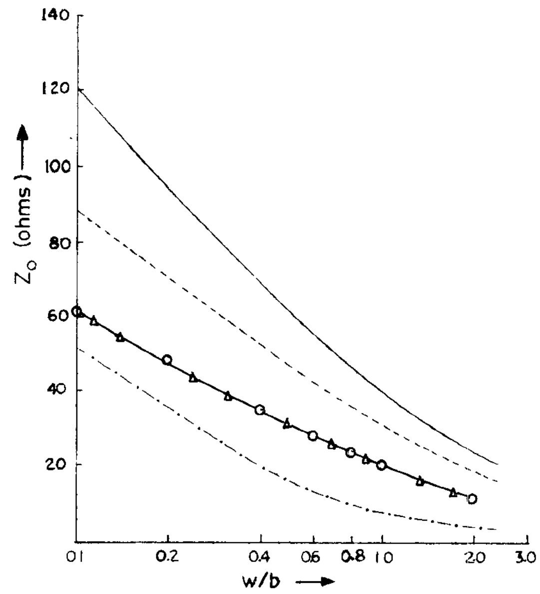

Fig. 2. Characteristic impedance as a function of strip width for $N=3$, $M=3$. ⊙⊙⊙⊙: see [12].

–·–·– $\epsilon_1 = 9.90, \epsilon_2 = 2.56, \epsilon_3 = 9.90, \epsilon_4 = 9.90, \epsilon_5 = 2.56, \epsilon_6 = 9.90$.

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continuous dielectric variation is used for the calculation of impedance data.

Results on characteristic impedance for the case of a stripline filled with six discrete dielectric layers of identical thickness made of two lossless dielectric materials are presented. Also, studies on the characteristics of a symmetric stripline where the filling of the dielectric medium is gradually varied from the bottom ground plane to the top ground plane have been carried out, and the results on characteristic impedance for this case are presented in Fig. 7. The limiting cases of stripline with (i) full air filling, (ii) the dielectric interface coincident with the center strip, and (iii) complete dielectric filling are compared and are in good agreement with those in the literature [3], [12]. Further, the results on impedance data for the case of continuous dielectric variation represented by Gaussian, inverted Gaussian, semicircular, inverted semicircular, parabolic, inverted parabolic, and cosine and inverted cosine on pedestal types of distributions are presented in Figs. 3–6.

II. Formulation

A. Analysis of Stripline with Discrete Multilayered Dielectrics Parallel to Ground Planes

Consider a stripline filled with layered dielectric media with their interfaces parallel to ground planes, as shown in Fig. 1(a). The conformal transformation which transforms one half of the structure, i.e., the region $P_1P_2P_3P_4P_5P_6P_7$ of Fig. 1(a) (z-plane), into the upper half of the t-plane (Fig. 1(b)) is found to be of the form [11]

$$Z = x + jy = \frac{2b}{\pi} \ln \left[\frac{\sqrt{1-mt^2} + \sqrt{m - mt^2}}{\sqrt{1-m}} \right] + jb \quad (1a)$$

where $2b$ is the thickness of the stripline structure. The following transform, which maps the upper half of the t-plane (Fig. 1(b)) into the rectangle shown in Fig. 1(c), is given by [11]

$$W' = u' + jv' = -\frac{F(\phi|m)}{K(m)} + \frac{K'(m)}{K(m)} \quad (1b)$$

where m is a constant and F and K correspond, respectively, to incomplete and complete elliptic integrals of the first kind with given argument and modulus. The value of the incomplete elliptic integral with complex argument in 1(b) can be expressed in terms of two incomplete elliptic integrals with real arguments and as [13]

$$F(\phi|m) = F(\beta|m) + jF(\nu|m_1) \quad (1c)$$

where $m_1 = (1-m)$, $\phi = \sin^{-1} t$, and $t = t_\gamma + jt_t$ with

$$t_\gamma = \frac{\sin\beta\sqrt{1 - m_1^2 \sin^2 \nu}}{\cos^2 \nu + m^2 \sin^2 \beta \sin^2 \nu}$$

$$t_t = \frac{\cos\beta \sin\nu \cos\nu\sqrt{1 - m^2 \sin^2 \nu}}{\cos^2 \nu + m^2 \sin^2 \beta \sin^2 \nu}.$$

Using the above transformations, the interfaces between the successive layers are conformally mapped to curved lines, as shown in Fig. 1(c). The conformal mapping of the boundary between two successive layers is valid, as it retains the angles of refraction at the interfaces.

The capacitance C_T of the parallel-plate configuration of Fig. 1(c) can be obtained by considering the parallel combination of the two capacitances C_L and C_R, where C_L and C_R are the capacitances due to rectangles $P_1P_4P_5P_6$

TABLE I
VARIATION OF CHARACTERISTIC IMPEDANCE (IN OHMS) FOR DIFFERENT COMBINATIONS OF DISCRETE DIELECTRIC LAYERS

Distribution	Number of layers selected		
	$N=5$, $M=5$	$N=10$, $M=10$	$N=15$, $M=15$
1. Gaussian $\epsilon_i=\epsilon_d \exp[-(2y_i-1)^2]$	65.20	66.01	66.31
2. Inverted Gaussian $\epsilon_i=\epsilon_d \exp[(2y_i-1)^2]$	59.24	58.71	58.52
3. Semicircular $\epsilon_i=\sqrt{\epsilon_d^2-(2y_i-1)^2}$	61.79	61.79	61.78
4. Inverted semicircular $\epsilon_i=\sqrt{\epsilon_d^2+(2y_i-1)^2}$	61.76	61.75	61.75
5. Parabolic $\epsilon_i=\epsilon_d-(2y_i-1)^2$	62.08	62.14	62.17
6. Inverted parabolic $\epsilon_i=\epsilon_d+(2y_i-1)^2$	61.49	61.43	61.41
7. Cosine on a pedestal $\epsilon_i=\epsilon_d+\cos(2\pi(y_i-0.5))$	62.81	63.10	63.16
8. Inverted cosine on a pedestal $\epsilon_i=\epsilon_d-\cos(2\pi(y_i-0.5))$	60.86	60.51	60.66

Strip width to ground plane ratio = 0.1.
Here, $y_i(=y/2b)$ is the position of the ith layer in the y-direction.

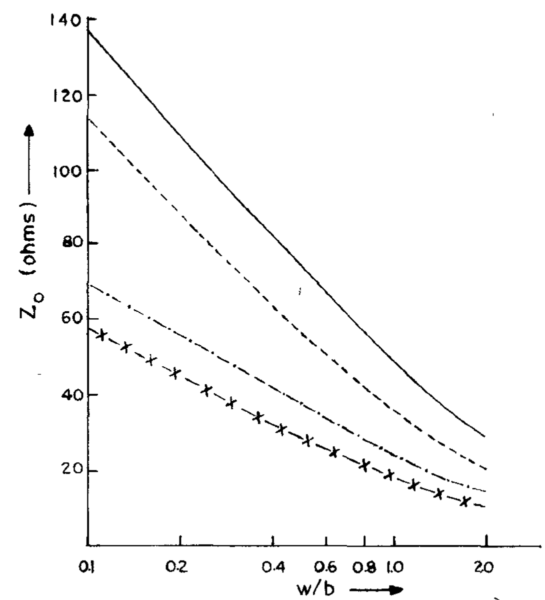

Fig. 3. Characteristic impedance as a function of strip width when the dielectric is distributed in a Gaussian form along the y-direction. —— Gaussian with $\epsilon_d=2.56$. --- inverted Gaussian with $\epsilon_d=2.56$. -·-·- Gaussian with $\epsilon_d=9.9$. -×-×- inverted Gaussian with $\epsilon_d=9.9$.

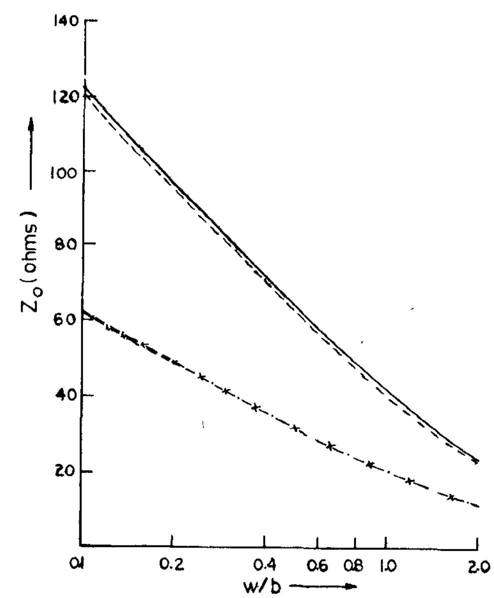

Fig. 4. Characteristic impedance as a function of strip width with circularly distributed dielectric constant along the y-direction. —— circular with $\epsilon_d=2.56$. --- inverted circular with $\epsilon_d=2.56$. -·-·- circular with $\epsilon_d=9.9$. -×-×- inverted circular with $\epsilon_d=9.9$.

and $P_1P_4P_3P_2$, respectively. The expression for the capacitance C_L due to the portion $P_1P_4P_5P_6$ can be derived by dividing it into a number of columns, each with an incremental width Δu^1 with height V_0, and integrating over the interval 0 to 1.

The incremental capacitance ΔC_L of the column $pqrs$ (Fig. 1(c)) is obtained by considering a series combination of small capacitances due to N dielectric layers; it is given by

$$\Delta C_L=\frac{\Delta u^1\epsilon_0}{\dfrac{V_0}{\epsilon_N}+\sum_{i=1}^{N-1} V_i^1[1/\epsilon_i-1/\epsilon_{i+1}]} \tag{2}$$

where V_i^1 is the position of the ith interface between the ith and $(i+1)$th dielectric layers in the W-plane, and ϵ_i is the relative dielectric constant of the ith layer.

In the above equation, as $\Delta u^1 \to 0$ the value of N (number of layers) approaches infinity; under this condition the summation in (2) becomes an integral within the limits of 0 to ∞. So the capacitance C_L is obtained by integrating (2) within the limits 0 to 1.

By following a similar procedure, the expression C_R for capacitance due to the portion $P_1P_4P_3P_2$ of Fig. 1(c) can be written as

$$C_R=\int_0^1 \frac{\epsilon_0\, du^1}{\dfrac{V_0}{\epsilon_{N+1}}+\sum_{i=1}^{M-1} V_{M-i}^1[1/\epsilon_{N+i}-1/\epsilon_{N+i+1}]}. \tag{3}$$

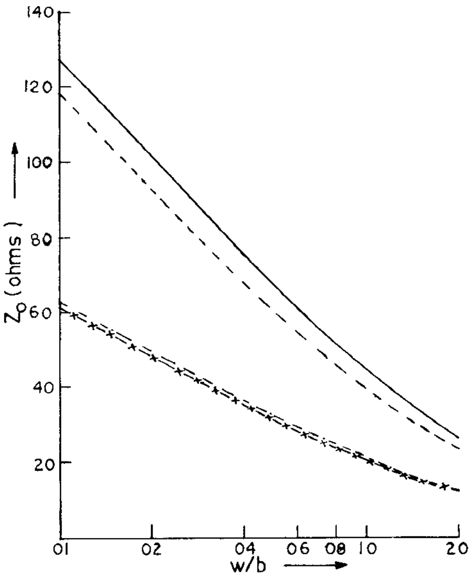

Fig. 5. Characteristic impedance as a function of strip width with parabolic dielectric distribution along the y-direction. —— parabolic with $\epsilon_d = 2.56$. ---- inverted parabolic with $\epsilon_d = 2.56$. —·—·— parabolic with $\epsilon_d = 9.9$. -X-X- inverted parabolic with $\epsilon_d = 9.9$.

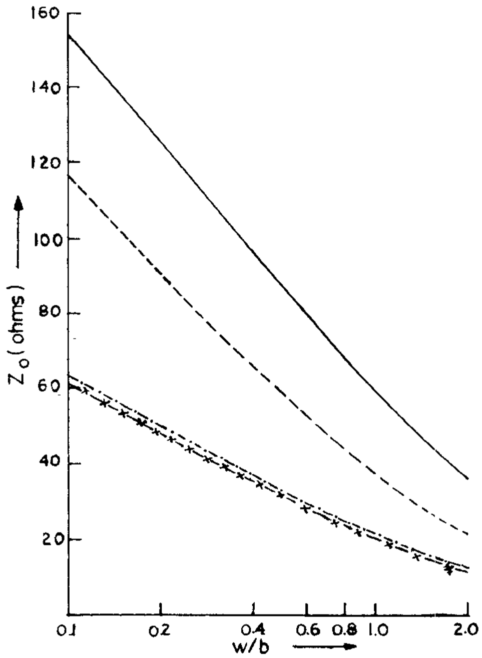

Fig. 6. Characteristic impedance as a function of cosine and inverted cosine on pedestal along the y-direction. —— cosine on pedestal with $\epsilon_d = 2.56$. ---- inverted cosine on pedestal with $\epsilon_d = 2.56$ —·—·— cosine on pedestal with $\epsilon_d = 9.9$. -X-X- inverted cosine on pedestal with $\epsilon_d = 9.9$.

The integrations appearing in the expressions for capacitances C_L and C_R are calculated by using the adaptive quadrature method [14]. For each value of u^1 selected, β is found from 1(b) and 1(c). For this value of β, the parameter ν is obtained by solving the transcendental equation

$$y_i - y = 0 \tag{4}$$

where y_i is the position of the ith dielectric layer in the z-plane (Fig. 1(a)). From the value of ν thus obtained from (4), V_i^1 is found using 1(b) and 1(c).

Since the stripline structure is symmetric about the y-axis, the total capacitance C is twice that of the capacitance of the parallel-plate configuration shown in Fig. 1(c) and is given by

$$C = 2C_T = 2(C_L + C_R). \tag{5}$$

From the value of the capacitance C thus obtained, the expression for the characteristic impedance is given by

$$Z_C = 30\pi v_0\sqrt{C_0/C} = Z_0/\sqrt{\epsilon_{\text{eff}}} \tag{6}$$

where $C_0 = 4\epsilon_0/v_0$, $v_0 = K'(m)/K(m)$, and ϵ_{eff} is the effective dielectric constant.

The variation of characteristic impedance as a function of strip width for the case of a stripline filled with six different dielectric layers ($N = 3, M = 3$), each having a thickness of $b/3$, is shown in Fig. 2. When the dielectric constant of all these six layers is identical, it corresponds to the case of uniform dielectric filling. The comparison of this case with the results available in the literature [12] has also been presented in Fig. 2.

B. Analysis of Stripline Filled with Medium Having Relative Dielectric Constant Varying Continuously in a Direction Perpendicular to Ground Planes

In this section, the evaluation of the characteristic impedance of stripline in the case where the relative dielectric

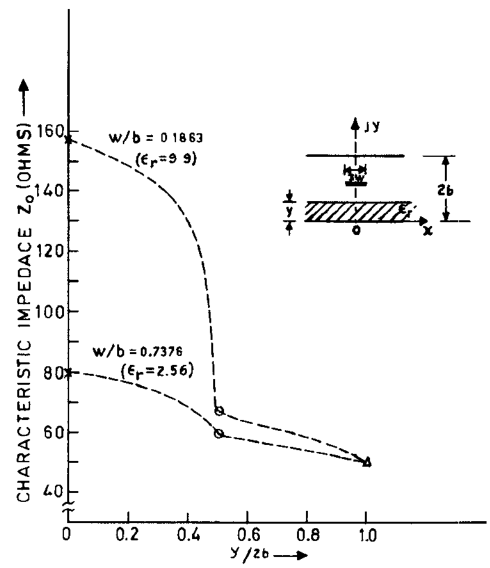

Fig. 7. Characteristic impedance as a function of the height of the dielectric layer along the y-direction. * *: $y/2b = 0.0$. ⊙⊙: $y/2b = 0.5$. △: $y/2b = 1.0$. Specific impedance values are available in the literature [3], [12].

constant of the medium is varying continuously in a specified manner in a direction perpendicular to ground planes is discussed. The analysis presented in the previous section is extended by considering a greater number of layers (ideally an infinite number of thin discrete layers) to realize the continuous variation of the relative dielectric constant of the medium along the y-axis.

Different types of distributions (for the variation of relative dielectric constant of the medium), viz., Gaussian, inverted Gaussian, semicircular, inverted semicircular, parabolic, inverted parabolic, and cosine and inverted cosine on a pedestal, are considered. The equations for the continuous variation of relative dielectric constant of these distributions are given in Table I. The equations representing different distributions are used to determine the relative dielectric constants of the finite layers. Using (2) and (3), the characteristic impedance of the stripline structure is calculated by increasing the number of discrete dielectric layers until the limit which gives the saturated value of impedance is achieved. The change in the characteristic impedance due to the increase in the number of layers for the above distribution has also been given in Table I. These distributions are defined with reference to $y_i = 0$ and are symmetric with respect to the central strip.

The variation of the characteristic impedance of the stripline filled with different distributions of relative dielectric constants discussed in Table I is presented in Figs. 3–6.

III. Results and Discussion

The analysis presented here is more general at low-frequency ranges and imposes no restriction on the number of dielectric layers and their corresponding widths. Hence, it is possible to realize any distribution of relative dielectric constant along the y-axis with a simple application of conformal transformation. This provides an additional advantage over other numerical methods, which may take considerable amounts of computer time, for structures having multilayered dielectric media. From the results presented in Fig. 7, it is observed that the change in the characteristic impedance is more or less the same for the case where the value of the relative dielectric constant laid around the center strip is high. The results presented for the case where there is a gradual filling of a single dielectric from the bottom to the top of the stripline for a given W/b ratio reveal that the value of the characteristic impedance is extremely sensitive to the filling of dielectric around the center strip. This sensitivity depends on the values of both W/b and the relative dielectric constant. The results on characteristic impedance for the limiting case calculated using the present formulation are compared with those in the literature for a stripline with discrete dielectric layers and are indicated in Fig. 7. Also, the results presented in Fig. 7 on stripline where the dielectric thickness is varied from zero to $2b$ (thickness of the stripline) show an excellent agreement with the values of impedance data calculated using the formulas available in the literature [3], [12] for the values of $y/2b = 0$, 0.5, and 1.0 with $W/b = 0.1868$ for $\epsilon_\gamma = 9.9$ and $W/b = 0.7376$ for $\epsilon_\gamma = 2.56$. The values of $y/2b$ equivalent to 0, 0.5, and 1.0, respectively, correspond to the cases of complete air filling, half dielectric filling, and complete dielectric filling of the stripline. For the case where the dielectric is distributed in the form of Gaussian and inverted Gaussian, the results reveal that the change in characteristic impedance for both of these cases remains the same for the values of w/b up to 2.0. It is also concluded that the effective dielectric constant (eq. 6) for these distributions is different by a significant amount. This can be observed from the results of Fig. 3. For the case of circular and parabolic variations of the dielectric constant (Figs. 4 and 5), the change in the effective dielectric constant between these distributions and their inverse distributions decreases with increasing values of ϵ_d. This implies that these distributions are identical to their corresponding inverse distributions with respect to their filling. But when the distribution is in the form of a cosine on a pedestal (the results of which are shown in Fig. 6), the change in characteristic impedance between this distribution and its inverse distribution is quite significant for low values of ϵ_d. From Table I, it is observed that most of the distributions can be realized even with ten ($N = 5, M = 5$) discrete dielectric layers.

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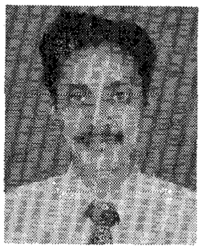

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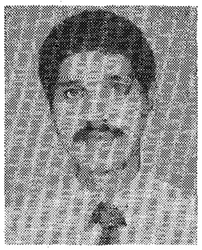

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